Self-organized criticality in atmospheric cascades

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We argue that atmospheric cascades can be regarded as example of the self-organized criticality and studied by using Lévy flights and nonextensive approach. It allows us to understand the scale-invariant energy fluctuations inside cascades in a natural way.

1. INTRODUCTION

It is well known that energy spectra of particles from atmospheric family events which are observed by the emulsion chambers at mountain altitudes are essentially following power-like dependence. On the other hand the occurence of power-law distributions is a very common feature in nature and it is usually connected with such notions as criticality, fractals and chaotic dynamics and studied using generalized (nonextensive) maximum entropy formalism characterized by a nonextensivity parameter q. Such formalism leads in a natural way to power-laws in the frame of equilibrium processes [1]. For nonequilibrium phenomena the sources of power-law distributions are self-organized criticality [2] and stochastic multiplicative processes [3]. In the former nonequilibrium systems are continuously driven by their own internal dynamic to a critical state with power-laws omnipresent. In the later power-law is generated by the presence of underlying replication of events. All these approaches can be unified in terms of generalized nonextensive statistics mentioned above (widely known as Tsallis statistics) [1].

We shall look therefore at the development of cascades from this point of view. To be more specific, we shall use the notion of Lévy flights as representatives of power-laws emerging from generalized statistics [1].

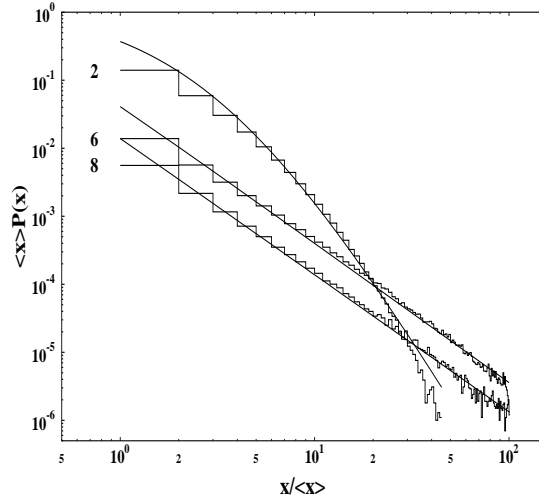## 2. MONTE-CARLO SIMULATION OF MULIPLICATIVE PROCESSES

Fig.1 Distributions $P(x)$ for multiplicative process, as given by eq. (1) (histograms), for different generation numbers $N = 2, 6$ and 8 are fitted by Lévy distribution (eq. (3)) (solid lines) with parameters $\alpha = 4.81, 2.05$ and 2.015, respectively.

Let us start with numerical (Monte-Carlo) consideration of the following model of multiplicative process:

$$x_{N+1} = \xi\, x_N, \qquad x_0 = 1, \tag{1}$$

where ξ is random number taken from the exponential distribution (to account for the essentially exponential scaled energy dependence of the single elementary interaction):

$$f(\xi) = \frac{1}{\xi_0} \exp\left(-\frac{\xi}{\xi_0}\right). \tag{2}$$

As was shown in [4], every exponential distribution with a fluctuating parameter results in the Lévy distribution (cf. Fig. 1), which in our case has the following form

$$P(x_N) = \frac{1}{\langle x_N \rangle} \cdot \frac{\alpha-1}{\alpha-2} \cdot \left[1 + \frac{1}{\alpha-2}\frac{x_N}{\langle x_N \rangle}\right]^{-\alpha}. \tag{3}$$
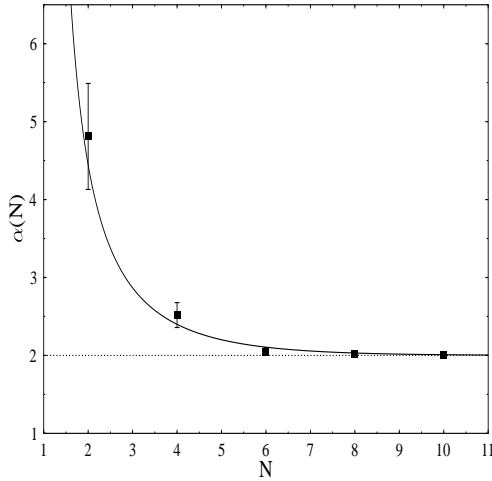

Fig.2 Dependence of the parameter α of the Levy distribution (3) on the generation number N. Solid line represents fit by the formula (4) with $c = 0.55$. Asymptotically $\alpha = 2$ (dotted line).

In our present case parameter α depends on the number of generations N and on the parameter ξ_0 which defines exponential distribution of the variable ξ (in the limit of large N dependence on ξ_0 effectively vanishes). As is illustrated in Fig. 2, dependence $\alpha(N)$ can be described by simple formula

$$\alpha = \alpha(N) = \frac{2}{1 - c^{N-1}} \tag{4}$$

where c is generation independent parameter which should be fitted (in our case $c = 0.55$).

If we use the canonical form of Lévy distribution as emerging from Tsallis statistics [1],

$$P(x) = \frac{2-q}{\chi}\left[1 - (1-q)\frac{x}{\chi}\right]^{\frac{1}{1-q}}, \tag{5}$$

where mean $\langle x \rangle$ is given by the parameter $\chi = (3-2q)\langle x \rangle$ and q is mentioned before nonextensive parameter such that $q = 1 + 1/\alpha$ [4] then

$$q = \frac{3}{2} - \frac{c^{N-1}}{2}. \tag{6}$$

Notice that for $N \to 1$ parameter $q \to 1$ (or, respectively, $\alpha \to \infty$), i.e., we are recovering the initial exponential distribution. In the limit $N \to \infty$ parameter q approaches value $q = 3/2$, which is limiting value available for q emerging from the normalization condition imposed on the probability distribution $P(x_N)$. In this limit $\alpha \to 2$. It is interesting to note that for $[x_N/\langle x_N \rangle] \cdot [1/(\alpha-2)] >> 1$, i.e., for x sufficiently large one gets power-like behaviour of $P(x_N)$:

$$P(x_N) \propto \left(\frac{x_N}{\langle x_N \rangle}\right)^{-\alpha}. \tag{7}$$

Actually such situation is reached reasonably fast, because $\langle x_N \rangle = \xi_0^N$ and α tends to its limiting value $\alpha = 2$ rather quickly with increasing number of generations N. In practice the equilibrium distribution $P(x_N) \propto x_N^{-2}$ is reached (for $x_N >> \langle x_N \rangle$) already for $N > 6$.

3. COMPARISON WITH EXPERIMENTAL DATA

There exists a number of experimental data from the emulsion chambers exposed at mountain altitudes, which are relevant for our approach and which we shall compare with now [5,6]. Although, as we have already mentioned, the energy spectra of particles from atmospheric family events they represent are roughly expressed by the power-law distributions, so far there was no analysis showing wheather it is true and to what extend. Our work is the first attempt in this direction as we show that they all can be described by the Lévy type spectra.
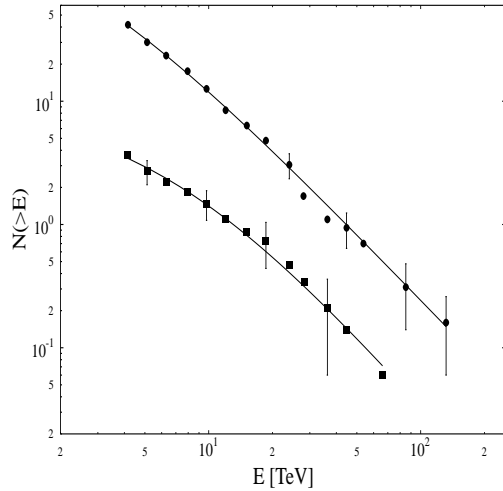
Fig.3 Integral energy spectra for gamma quanta (circles) and hadrons (squares) in families registered at Mt. Kambala are fitted by Lévy distributions (solid lines).
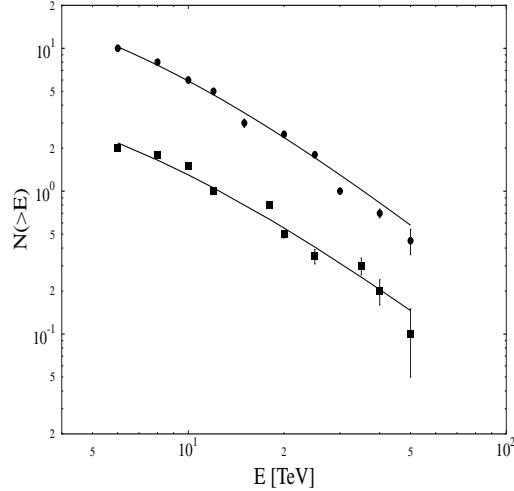

Fig.4 Integral energy spectra for gamma quanta (circles) and hadrons (squares) in families registered at Mt. Pamir are fitted by Lévy distributions (solid lines).

Integral spectra $N(> E)$ from Kambala experiment (at $520\,\mathrm{g/cm^2}$) [5] are shown in Fig. 3 where they are fitted using parameters $\alpha - 1 = 1.8$ (for gamma families) and $\alpha - 1 = 2.0$ (for hadronic families). Notice that hadronic component is little "younger" (higher α means smaller N, cf. Fig. 2) than the electromagnetic component.

Families observed deeper in the atmosphere (what means higher N) have smaller α, as expected, cf. Fig. 4, where $N(> E)$ from Pamir experiment (at $600\,\mathrm{g/cm^2}$) [6] are presented. Here the corresponding parameters $\alpha - 1$ are equal to 1.8 and 1.7 for gamma and hadronic components, respectively.

It is evident from Figs. 3 and 4 that the observed gamma-hadron families are already reaching their quasi-equilibrium states. It should be noticed that most of the families registered come not from a single nuclear interaction but they usually contain particles which are decascadents of several (~ 7 at $\Sigma E > 100$ TeV) genetically connected nuclear interactions [7]. Our analysis indicates that the average number of cascade generations leading to the observed distributions is about $N \simeq 3$.

4. SUMMARY AND DISCUSION

We have provided description of cascade processes encountered in cosmic ray emulsion experiments at high altitudes, which is based on the notion of Lévy distrubutions (which, in turn, originate from the nonextensive statistics described by parameter q [1,4]). In this way we have found that the observed distributions are to a high accuracy power-like, but at the same time we are able to account for the small deviations from the exact power-like behaviour. They are, in our approach, directly connected with the finite number N of the cascade generations in the way provided by formulas (4) or (6).

The results presented before should be confronted with the result of pure nuclear origin, which is the case of the so called "halo" events with strong concentration of particles in the central region. Fig. 5 shows such a case exposing integral energy distribution of shower cores (recorded at distance $R < 0.65$ cm from the energy-weighted center) obtained in a "halo" event P06 registered at Chacaltaya [8]. In this

case we observe almost exponential distribution ($\alpha - 1 = 16.0$ in this case, what translates to $q = 1.06$) with no influence of multiplicative processes.
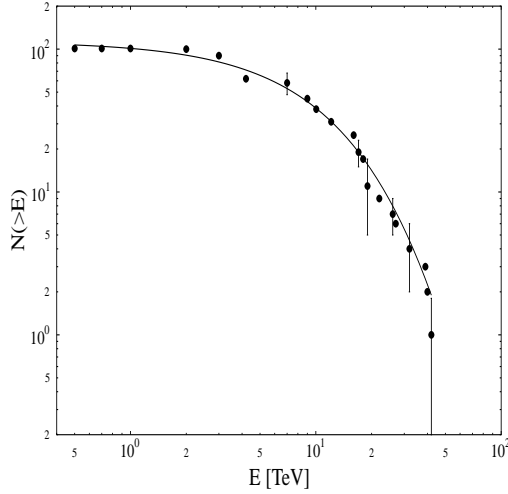

Fig.5 Integral energy spectra for shower cores from halo event (circles) registered at Chacaltaya is fitted by Lévy distribution (solid lines).

Finally, it should be noticed that there exists a number of special cases more-or-less directly connected with our results:
(a) Energy transport equation [9] for cascade processes gives

$$N(E,t) = N_0 E^{-(s+1)} \exp\left(-\frac{s-1}{s+1} \cdot \frac{t}{t_0}\right) \qquad (8)$$

as number of particles with energy E at depth t. Notice that for the age parameter $s \to 1$ one approaches equilibrium spectrum $N(E) = N_0 E^{-2}$ with the slope $\alpha = 2$.
(b) One observes strong intermittent behaviour in the families [10], which results from the fluctuations in the atmospheric cascades alone and is not sensitive to details of the elementary interactions.
(c) One observes multifractal behaviour in the cascades with fractal dimension D_N at each cascade stage N where $P(x) \propto x^{D_N}$ [11]. For each successive emission the distribution should be more inhomoheneous, i.e., $D_N > D_{N+1}$.
(d) Self-organized criticality [2] to which all cascades considered here bear close resemblance because of the Zipf's law [12], which predicts power-law behaviour of the type seen in eq.(7) (with $\alpha = 2$).

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